**SS3 FURTHER MATHEMATICS**

**SIGMA TERM (SECOND TERM)**

SCEME OF WORK

WEEK(S) TOPIC

1. Review of first term`s examination questions and STATIC: Moment of force (2 and 3 forces) acting at a point.
2. STATIC: (i) Polygon of forces (ii) Resolution of forces of friction.
3. MODELLING: (i) introduction to modeling (ii) Dependent and independent variables in Mathematical modeling (iii) examples of some models
4. MODELLING: (I) Construction of model (ii) Methodology of modeling (iii) Application to physical, biological, social and behavioral services.
5. GAMES THEORY: (I) Introduction to games theory (ii) Description of types of games.
6. GAMES THEORY: (i) solution of two person (ii) Zero sum games using pure and minimized strategies (iii) matrix games.
7. To 12 revision and Mock Examination

**WEEK 1**

REVISION AND STATICS

LINEAR INEQUALITIES IN ONE VARIABLE

Find the solution sets of the following inequalities:

1. 3x + 2 x + 4 (b)

Solution

1. 3x + 2

3x x 4 2

2x 2 x

X 1

EXAMPLE

LOGARITHM

Solve the given equation (= 0

Solution

( + 8 = 0 ) = -8

= = = cross multiply

= = , x = 9.

STATIC

STATIC: is defined as a branch of Mechanics which deals with the study of bodies at rest acted upon by forces.

MOMENT OF A FORCE ACTING AT A POINT.

Moment of a force about a given point is the turning effect of a force about an axis through a point . i.e. the product of the force and the force arm.

= x and

Moment of a force is a vector quantity and its unit is Nm.

PRINCIPLE OF MOMENT

The principle of moment states that if a system is in equilibrium under the actions of any numbers of the coplanar forces. Then the sum of their moment about any point in the plane of the coplanar forces is zero.

Let R be the resultant of two coplanar forces about any point A, then = i.e. the sum of the moment of two coplanar forces about a point in the plane of the force is equal to the moment of the resultant of the two forces about the same point.

If a system of coplanar forces is in equilibrium, then the sum of the clock-wise moment is equal to the anti-clock-wise moment about the same point in the plane.

EXAMPLE

A uniform plank X and Y is 16m long and has a mass of 18kg. The plank rests on two supports at X and Y and a load of mass 8kg is placed on the plank at a point Z, 6m away from X. Calculate the reactions of the support at X and Y on the plank. (Take g = ).

SOLUTION

16m

8m

8m

6m

Reaction X

Reaction Y

8kg 18kg

Let be the reaction at X and be reaction at Y

Taking moment about the point X

x 16 = 8g x 6 + 18g x 8

x 16 = 48g + 144g = 192g

192 x 9.8 = 1881.6

= = 117.6

= 118N.

x 16 = 8g x 10 + 18g x 8

80g + 144g = 224g

= =

= 137.2 137N

: work the following questions:

3, exercise 13, page 205 questions: 7, 8 18 and 23

5B, Questions: 1, 4 7, 18 and 19

**WEEK 2**

STATIC FORCES

POLYGON AND RESULUTION OF FORCES

FORCES: Force is that action that tends to change the state of body at rest or uniform motion in a straight line.

POLYGON OF FORCES: Is a broken line that is constructed to determine the resultant vector (geometric sum) of a given system of forces.

RESOLUTION OF FORCES

Resolution of forces is when a given force is been resolved into two parts. i.e. x-axis called horizontal component and y-axis called VERTICAL COMPONENT.

A C

R

x

O y B N

By cosine rule, 2xy

If R is the inclination of R to B, then

= =

PY P

PX

If a force p acts at a point of angle to the x-axis. i.e.

= P

= P

= = ).

EXAMPLE Resolution of forces

A force P is 230N, it is inclined to the horizontal at an angle of . Find the horizontal and vertical components of P.

SOLUTION

P=230N

25º

P­x

= P = 230

= 230 x 0.9863 = 208.5N

Therefore, the horizontal component force is 209N.

= p = 230

= 230 x 0.4226 = 97.2N

Therefore, the vertical component force is 97N.

ASSESSMENT: Work the following questions from New Further Mathematics Project 3: Page 186, exercise 12. Questions: 1,2,3, 5,7 and 8.

Page258 revision exercise chapter 12, questions: 13, 17, 20 and 23.

**WEEK 3**

MODELLING (I)

DEFINITION OF MODELLING

Modelling in Mathematics is the application of mathematics to real life situations.

USES OF MODELLING IN MATHEMATICS

* Industry
* Commerce
* Government
* Weather forecasting
* Health institution
* Educational institutions etc.

AREAS MOSTLY USED OF MATHEMATICAL MODELLING

* + Predictions of population growth
  + Financial business (stock trading)
  + War and conflicts
  + The spread of deceases and epidemics.

MODEL: Model in Mathematics is a concept that represents a real life situation. This is developed to help in the understanding of the physical phenomena, such as: to make observations, collections of data and to carry out an experiment.

EXAMPLES OF MATHEMATICAL MODEL

1. Theory of Malthusian population growth. i.e. ( the rate of food is growing arithmetically while the population is growing geometrically)
2. House hold financial budget
3. Geographical projection of an area or region on earth surface.(planning).
4. The behavior of consumers choice in the prices of commodities.

VARIABLES

There are basically six variables in Mathematical modeling. i.e.

Output variables, input variables, random variables, decision variables, state variables and exogenous variables.

DEPENDENT VARIABLE: This represents the output or outcome whose variation is being studied to see if and how much it varies as the independent variables vary in the simple stochastic (randomly determined) linear model. i.e. = a + + . The term is the ith value of the dependent variable , is ith value of the independent variable and is the error and contains the variability of the dependent variable not explained by the independent variable.

INDEPENDENT VARIABLES: Are the inputs or causes (potential reasons for variation). i.e. x is input and y is output. Y = f(x). for multiple independent variables z = f(x,y). = a + + + …+ + . Where n is the number of independent variables.

ASSESSMENT: New Further mathematics project 3, page 225, exercise 15, question 3, 4, 6 and 7.

**WEEK 4**

MODELLING (II)

EXOGENOUS VARIABLE: These are independent variable that affects a model without being affected by it, and whose qualitative characteristics and method of generation are not specified by the model builder. An exogenous variable is used for setting arbitrary external conditions and not in achieving a more realistic model behavior.

CONSTRUCTION OF MODELLING AND IT`S APPLICATION.

CONSTRUCTION

Let x(t) be the amount of substance still present at time *t.* From differential calculus is the time rate of change of decay. Then, the time rate of change is proportional to x. i.e. = kx…..(i)

EXAMPLE

Observations have shown that the rate of change of the atmospheric pressure. Given that the pressure at 6000m above the sea level is half its value at sea level. Find the formula for the pressure at any height h.= Inp = kh +c

P = P = , (A = )

= A…..(i)

When the height (h) is zero

= A = A

Therefore, at height (h) = 6000m

P =

= P =

= 1 (formula for the pressure at any height h).

ASSESSMENT: work out the following questions: New Further Mathematics project 3, page 224, Exercise 15. Questions 3 ,9 and 10

**WEEK 5**

GAMES THEORY (I)

Is a set of concepts aimed at decision making in situations of competitions or conflicts (as well as of co-operation and interdependence) under specified rules. It is a decision theory which is applicable to competitive situations.

COMPETITIVE GAME: this is the competitive situations which occur between two or more persons.

PROPERTIES OF A COMPETITIVE GAME

* The number of participants is stated or finite.
* Each action of operations of individuals is certain
* The choices of players are made simultaneously, in other for the participants choices to be personal to the others until he makes his decision.
* The players of the games must know all the courses of actions available to but must not know which of these courses will be chosen. TYPES OF GAMES
* One-person games
* Two-person games
* Two-person zero sum games
* N-person games.

ONE-PERSON GAMES

This is a game that consists of one player. There is no competition or conflicts in this, because the interest of the only player is at stake. i.e. solitaire game.

TWO-PERSON GAMES

This involves two players who have conflicting interest. Each players has several possible choices open to them at each play of the game. E.g. checkers, chess etc.

N-PERSON GAMES

This involves unspecified numbers of players. E.G. POKER (board and card games), monopoly game etc.

**WEEK 6**

GAMES THEORY (II)

PAYOFF (OUTCOME) MATRIX

This is an expression of the first law of decision science. Each row represents one action that the decision maker might or might not freely choose to perform while each column represents a possible state of nature.

Given two player A and B, if the outcome of is positive, then it represents a gain to player A and a loss to player B. But if is negative, the it represents a loss to A but a gain to B

PLAYER A

1 2 3… j n

1 …

2 ….

i …

m ….

PLAYOFF MATRIX A

PLAYER B

1 2 3 j n

1 …

2 …

i …

M …

PLAYOFF MATRIX OF B

MAXIMINI AND MINIMAXI PRINCIPLES.

THE MAXIMIN PRINCIPLE: This is a decision rule used in game theory to maximize the minimum guaranteed gains of player A

The maximum of these minimum gains is called the MAXIMIN VALUE and the corresponding alternative course of action is called the MAXIMIN STRATEGY.

MINIMAX PRINCIPLE

This is a decision rule in game theory that is used to minimize the maximum losses.

The minimum of these maximum losses is called the MINIMAX VALUE and the corresponding alternative course of action is called MINIMAX STRATEGY.

SADDLE POINT.

This is when the maximin value is equal to the minimax in a game.

EXAMPLE

Solve the game whose payoff matrix is given by

Player A

Player X 1 4 1

0 -3 -2

1 6 0

SOLUTION

Player Y = Row minima i.e. 1, -3, 0.

= 1. = -3 and = 0

Player X = column maxima i.e. 1, 6, 1.

= 1. = 6. = 1.

Maxi (minimum) = max (1, -3, 0 ) = 1

Mini (maximum) = min(1, 6, 1) = 1

Therefore, maximin = 1

Minimax = 1.

ASSESSMENT: New further Mathematics project 3 by tuttuh-Adegun etal. Page 236, Exercise 16 questions 1a and b. 2, 3 and 4.

WEEK 7 – 12 MOCK EXAMINATION AND WAEC EXAMINATION.